DNA Codes Over the Ring $F_2 + UF_2 + VF_2$

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Abstract

This work examines the structural properties of even-length cyclic DNA codes in the ring $F_2 + UF_2 + VF_2$. We analyze cyclic codes over an even-length ring R, examining their adherence to both Use practical instances to demonstrate the reverse constraint and reverse complement constraint limitations.

Keywords: DNA Cyclic Codes, Reversible constraint, Watson Crick Complement, Hamming distance.
Subject Classification: 94B15

Introduction

Deoxyribonucleic acid (DNA) contains the genetic instructions that control cell structure and function. The cell's life depends on its ability to operate, reproduce, create, repair, and execute other necessary activities. Information is stored in living creatures for lengthy periods of time. DNA is the fundamental programming unit of life. DNA is made up of four nucleotides: Adenine (A), Guanine (G), Thymine (T), and Cytosine (C). Throughout the article, we employ (A, G, T, C). Hybridization is the process by which each nucleotide on a DNA strand forms a double helix by forming hydrogen bonds with its complementary base. For example, A is the complement of T and G is the complement of C.

A single DNA strand consists of the letters A, G, T, and C, with chemically distinct polar terminals at the 5- and 3-ends. These strands together form a double helix. This combination is This was done by following the Watson-Crick model.

DNA strands pair in reverse order. Consider the Watson-Crick complementary (WCC) strand as $3' - ACTTAGAA - 5'TTCTAAGT - 3'$. Adleman.L was the first to show the use of DNA for computing. [3] in 1994, by solving a Hamilton path problem with DNA strands. His approach was based on hybridization, a fundamental idea in DNA computations. In [1, 2], Adleman expressed interest in DNA computing and the Pioneer. We examined an NP-complete computational problem with DNA molecules in a test

1

tube. Hammons et al. [10] used the Gray map to generate non-linear binary codes for Z_4.

Linear codes over finite rings gained popularity in algebraic coding theory after the introduction of gray maps in the early 1970s. In late twentieth century algebraic coding theory, linear code over finite rings was a major focus. This study of finite rings was inspired by The grey maps have been completed. Siap et al. [13] studied DNA codes using the finite ring $F_2(u)/ < u^2 - 1 >$, which has four components.

The study [7] examines cyclic codes over the ring $Z_4 + WZ_4$. Yildiz and Siap [6] studied DNA codes across the ring $F_2(u)/ < u^4 - 1 >$, which includes 16 elements. For further information on cyclic DNA coding, see [8, 9, 12]. Several publications have Several ways were studied to create a library of DNA codewords that are unlikely to form undesired connections through hybridization [4,5,11].

This article begins with basic material, then discusses the key conclusions of the reversible constraint, and lastly addresses the reversible complement constraint for DNA codes over the ring $F_2 + UF_2 + VF_2$.

Preliminaries

Definition 1 [7] Let $\sum DNA = A, G, T, C$. The set of M distinct DNA sequences each of length n such that the Hamming distance d_H between any two DNA sequences is d_H called DNA code. We denote the minimum Hamming distance d_H= $\{\ mind_H(x, y) : x \neq y, x, y \in DNA\}$. Mathematically, denoted as DNA $(n, M, d_H) \subseteq \sum^n DNA = (A, T, G, C)^n$.

Definition 2 [7] A linear code C of length n over R is said to be reversible $X^r \in C$ for all X$\in C$, complement if $X^c \in C$ for all $X \in C$ and reversible-complement if $X^{rc} \in C$ for all $X \in C$.

Lemma 3 [11]

Let $f(x), m(x)$ be any two polynomials in R with $deg(f(x)) \geq deg(m(x))$, then

[i] $[f(x)m(x)]^* = f^*(x)m^*(x)$.

[ii] $[f(x) + m(x)]^* = f^*(x) + x^{degf^{(x)-degm(x)}}m^*(x)$

Lemma 4 [9]

For any $a \epsilon R$ we have $a + \overline{a} = 1 + u$

Lemma 5 [9]
For any $a, b, c \epsilon$R then,
(i) $\overline{a+b} = \overline{a} + \overline{b} + 3(1+u)$
(ii) $\overline{a+b+c} = \overline{a} + \overline{b} + \overline{c} + 2(1+u)$
Lemma 6 [9]
For any $a \epsilon R$ then we have
(i) $\overline{2a} + 3(1+u) = 2a$
(ii) $\overline{a} + 3(1+u) = 3a$

Reversible Constraint

Reversible constraints in the context of DNA refer to processes or modifications within the genetic material that can be undone or altered under specific conditions. These constraints play a crucial role in maintaining genetic stability and regulating gene expression. Examples of reversible constraints include DNA repair mechanisms that correct mutations, epigenetic modifications such as DNA methylation that can be reversed to regulate gene activity, and the use of gene-editing technologies like CRISPR to introduce reversible genetic changes. This ability to reverse or regulate changes in the genetic code is essential for cellular adaptability, development, and response to environmental factors.

Theorem 1
Let $C = x_1(a) + 2x_2(a) + 2um(a)$ with $x_2(a)|x_1(a)|a^n - 1$ be a cyclic code of even length n over R, then C is reversible if and only if
(i) $x_2(a)$ is self reciprocal
(ii) $a^i x_2^*(a) = x_2(a)$ and
$a^j m^*(a) = m(a)$ or $x_2(a)|(2a^j m(a) + 2m(a))$
where $i = degx_2(a) - degx_1(a)$, $j = degx_2(a) - degm(a)$
Proof
Suppose $C = x_1(a) + 2x_2(a) + 2um(a)$ is reversible over R, then C is reversible over F_2 gives,
$(x_1(a) + 2x_2(a) + 2um(a))^* = x_1^*(a) + 2a^i x_2^*(a) + 2ua^j m^*(a)$
$= x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a)$ (1)
$= x_1(a) + 2x_2(a) + 2um(a)x(a)$ (2)
where $i = degx_2(a) - degx_1(a)$ $j = degx_2(a) - degm(a)$
Comparing the degrees of (1) and (2) we get,
$x(a) = c$ where $c \epsilon R$ then,
$x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a) = c.x_1(a) + c.2x_2(a) + c.2um(a)$ (3)
$2u.x_1(a) + 2u.2a^i x_2^*(a) + 2u.2ua^j m^*(a) = 2u.c.x_1(a) + 2u.c.2x_2(a) + 2u.c.2um(a)$
$2ux_1(a) = c.2ux_1(a)$ that implies $c = 1, 1+u, 1+2u (or) 1+3u$ if $c = 1$

3

$(3) \Rightarrow x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a) = x_1(a) + 2x_2(a) + 2um(a)$

$$2a^i x_2^*(a) + 2ua^j m^*(a) = 2x_2(a) + 2um(a) \quad (4)$$

$u.2a^i x_2^*(a) + u.2ua^j m^*(a) = u.2x_2(a) + u.2um(a)$

$2ua^i x_2^*(a) = 2ux_2(a)$ as $u^2 = 0$ implies that

$a^i x_2^*(a) = x_2(a)$, where $i = degx_2(a) - degx_1(a)$

$a^j m^*(a) = m(a)$ where $j = degx_2(a) - degm(a)$

for $c = 1 + u$

$(3) \Rightarrow$

$x_1(a)+2a^i x_2^*(a)+2ua^j m^*(a) = (1+u)x_1(a)+(1+u)2x_2(a)+(1+u)2um(a)$

$$x_1(a)+2a^i x_2^*(a)+2ua^j m^*(a) = x_1(a)+ux_1(a)+2x_2(a)+2ux_2(a)2um(a) \quad (5)$$

Multiplying u both side of (5) we get,

$u.x_1(a)+2ua^i x_2^*(a)+2u^2a^j m^*(a) = u.x_1(a)+u^2x_1(a)+u^2x_1(a)+2ux_2(a)+ 2u^2x_2(a) + 2u^2m(a)$

$2ua^i x_2^*(a) = 2ux_2(a)$

$a^i x_2^*(a) = x_2(a)$

$2ua^j m^*(a) + 2um(a) = ux_1(a) + 2ux_2(a)$

Since $x_2(a)|x_1(a)$ then we have,

$(2a^j m^*(a) + 2m(a)\epsilon x_2(a)$

$\therefore x_2(a)|2a^j m^*(a) + 2m(a))$

for $c = 1 + 2u$

$(3) \Rightarrow x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a) = (1 + 2u)x_1(a) + (1 + 2u)x_2(a) + (1 + 2u)2um(a)$

$$x_1(a)+2a^i x_2^*(a)+2ua^j m^*(a) = x_1(a)+2ux_1(a)+2x_2(a)+2um(a) \quad (6)$$

$u.x_1(a) + 2ua^i x_2^*(a) + 2u^2m^*(a) = ux_1(a) + 2u^2x_1(a) + 2ux_2(a) + 2u^2m(a)$

$2ua^i x_2^*(a) = 2ux_2(a)$

$a^i x_2^*(a) = x_2(a)$

$2a^j m^*(a) + 2m(a) = 2x_1(a)$

$\therefore x_2(a)|2a^j m^*(a) + 2m(a)$ as,

$x_2(a)|x_1(a)$ similarly for $c = 1 + 3u$

$(x_1(a) + 2x_2(a) + 2um(a))^* = x_1^*(a) + 2a^i x_2^*(a) + 2ua^j m^*(a)$

$= x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a)$

$= x_1(a) + 2x_2(a) + 2um(a)c\epsilon C$

where $c = 1, 1 + u, 1 + 2u(or)1 + 3u$

$i = degx_2(a) - degx_1(a)$, $j = degx_2(a) - degm(a)$

Hence C is reversible.

4

ISSN : 1671-6841

Theorem 2

Let $C = x_1(a)+2x_2(a)+2um(a), ux_3(a)+2ux_4(a)$ with $x_2(a)|x_1(a)|(a^n-1)$ and $x_4(a)|x_3(a)|(a^n-1)$ in $\frac{R[x]}{<a^n-1>}$ be a cyclic code of even length n over R. Then C is reversible if and only if,

(i) $x_1(a)$ and $x_3(a)$ are self reciprocal,

(ii) $a^i x_2^*(a) = x_2(a)$ and

$x_4(a)|(2a^j m^*(a) + 2m(a))$ (or)

$x_4(a)|(2a^j m^*(a) + 2m(a) + 2x_2(a))$

where $i = degx_1(a) - degx_2(a)$

$j = degx_1(a) - degm(a)$.

Proof

Let $C = x_1(a) + 2x_2(a) + 2um(a), ux_3(a) + 2ux_4(a)$.

$(x_1(a) + 2x_2(a) + 2um(a))^* = x_1^*(a) + 2a^i x_2^*(a) + 2ua^j m^*(a)$

$= x_1(a)+2a^i x_2^*(a)+2ua^j m^*(a)$ (1)

$= (x_1(a) + 2x_2(a) + 2um(a))x(a) + u(x_3(a) + 2x_4(a))b(a)$

$= (x_1(a)+2x_2(a)+2um(a))x(a)+ux_4(a)b(a)$ (2)

Since,

$x_4(a)|x_3(a)$

Here $i = degx_1(a) - degx_2(a)$

$j = degx_1(a) - degm(a)$

Comparing the degree of (1) and (2) we have $x(a) = c, c\epsilon R$

$x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a) = c.x_1(a) + c.2x_2(a) + c.2um(a) + u(x_3(a) + 2x_4(a)b(a))$ (3)

$2ux_1(a)+2u.2a^i x_2^*(a)+2u.2ua^j m^*(a) = 2u.cx_1(a)+2u.c2x_2(a)+2u.c.2um(a)+ 2u.u(x_3(a) + 2u.2x_4(a)b(a)$

$2ux_1(a) = 2u.cx_1(a)$

That implies, $c = 1, 1 + u, 1 + 2u$ or $1 + 3u$

If $c = 1$ then by (3) we have,

$x_1(a)+2a^i x_2^*(a)+2ua^j m^*(a) = x_1(a)+2x_2(a)2um(a)+u(x_3(a)+2x_4(a)b(a)$ (4)

$ux_1(a)+2ua^i x_2^*(a)+2u.ua^j m^*(a) = ux_1(a)+u2x_2(a)+u2um(a)+u.u(x_3(a)+ u2x_4(a)b(a)$

$2ua^i x_2^*(a) = 2ux_2(a)$ as $u^2 = 0$

That implies that $a^i x_2^*(a) = x_2(a)$

where $i = degx_1(a) - degx_2(a)$

Again using that result on (4) we get,

$2ua^j m^*(a) + 2um(a) = u(x_3(a) + 2x_4(a))b(a)$,

Since,

$x_4(a)|x_3(a)$ then we have,

$(2a^j m^*(a) + 2m(a))\epsilon F_2(a)$

$x_4(a)|(2a^j m^*(a) + 2m(a))$

5

For $c = 1 + u$ then by (3) we get,

$x_1(a) + 2a^i x_2^*(a) + 2ua^j m^*(a) = (1 + u)x_1(a) + (1 + u)2x_2(a) + (1 + u)2um(a) + u(x_3(a) + 2x_4(a))b(a)$

$$= x_1(a)+ux_1(a)+2x_2(a)+u2x_2(a)+2um(a)+u(x_3(a)+2x_4(a))b(a) \quad (5)$$

$a^i x_2^*(a) = x_2(a)$

Again using that result on (5) we get,

$2ua^j m^*(a) + 2um(a) + 2ux_2(a) = ux_1(a) + u(x_3(a) + 2x_4(a))b(a)$

Since $x_4(a)|x_3(a)|x_1(a)$ then, we write

$(2a^j m^*(a) + 2m(a) + 2x_2(a))\epsilon F_2(a)$

$x_4(a)|(2a^j m^*(a) + 2m(a) + 2x_2(a))$

Similarly for $c = 1 + 2u$ and $1 + 3u$ we get the same result as $c = 1, 1 + u$ respectively.

We can easily proved the converse part,

$\therefore$ C is reversible.

The above theorem illustrated with the following example,

$(x^{12} - 1) = (x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)(x^6 + 1)$.

Let $C = x_1(a) + 2x_2(a) + 2um(a), ux_3(a) + 2ux_4(a)$. Where $x_1(a) = x_2(a) = h_2$ $x_3(a) = h_1, x_4(a) = 1$ and $m(a) = 0$ we check that $x_1(a)$ and $x_3(a)$ are self reciprocal. $a^i x_2^*(a) = x_2(a)$ and $x_4(a)|(2a^j m^*(a) + 2m(a))$ the hamming distance 12 and a DNA code of length 12 are given in Table - 1.

6

A DNA code of length 12 obtained from

AAAAAAAAAAAA	ATAATAATAATA	ACAACAACAACA	AGAAGAAGAAGA
AATAATAATAAT	ATTATTATTATT	AGTAGTAGTAGT	ACTACTACTACT
AAGAAGAAGAAG	ATGATGATGATG	AGGAGGAGGAGG	ACGACGACGACG
AACAACAACAAC	ATGATGATGATG	AGCAGCAGCAGC	ACCACCACCACC
TAATAATAATAA	TTATTATTATTA	TGATGATGATGA	TCATCATCATCA
TATTATTATTAT	TTTTTTTTTTTT	TGTTGTTGTTGT	TCTTCTTCTTCT
TAGTAGTAGTAG	TTGTTGTTGTTG	TGGTGGTGGTGG	TCGTCGTCGTCG
TACTACTACTAC	TTCTTCTTCTTC	TGCTGCTGCTGC	TCCTCCTCCTCC
GATGATGATGAT	GTAGTAGTAGTA	GGAGGAGGAGGA	GCAGCAGCAGCA
GAAGAAGAAGAA	GTAGTAGTAGTA	GGAGGAGGAGGA	GCAGCAGCAGCA
GATGATGATGAT	GTTGTTGTTGTT	GGTGGTGGTGGT	GCTGCTGCTGCT
GAGGAGGAGGAG	GTGGTGGTGGTG	GGGGGGGGGGGG	GCGGCGGCGGCG
CAACAACAACAA	CTACTACTACTA	CGACGACGACGA	CCACCACCACCA
CATCATCATCAT	CTTCTTCTTCTT	CGTCGTCGTCGT	CCTCCTCCTCCT
CAGCAGCAGCAG	CTGCTGCTGCTG	CGTCGTCGTCGT	CCGCCGCCGCCG
CACCACCACCAC	CTCCTCCTCCTC	CGCCGCCGCCGC	CCCCCCCCCCCC

Table-1

Reversible Complement Constraint

The reversible complement constraint is a fundamental principle in various fields, including bioinformatics, combinatorial design, and coding theory. It ensures that a sequence remains valid when both reversed and complemented, preserving its structural or functional properties. In DNA analysis, this constraint is crucial for palindromic sequences and genetic pattern recognition, while in combinatorics and data encoding, it aids in designing robust codes and symmetric structures. By maintaining consistency under these transformations, the reversible complement constraint plays a key role in error detection, cryptography, and biological sequence analysis.

Theorem 3

Let $C = x_1(a) + 2x_2(a) + 2um(a)$ with $x_2(a)|x_1(a)|(a^n - 1)$ be a cyclic code of even length n over R. Then C is a reverse complement if and only if,

(i) $x_2(a)$ is self reciprocal,

(ii) $a^i x_2^*(a) = x_2(a)$ and

$a^j m^*(a) = m(a)$ or $x_2(a)|(2a^j m^*(a) + 2m(a))$

where $i = degx_2(a) - degx_1(a)$, $j = degx_2(a) - degm(a)$

7

Proof

Let C be a Cyclic Code of the WCC of it should also be in C by lemma 5 we have, $3(\overline{0,0,......0}) = 3(1+u,1+u,....1+u) = 3(1+u)\frac{1-a^n}{1-a}\epsilon C$

$x_1(a) = 1 + f_1'(a) + ... + f_{s-1}'a^{s-1} + a^s$

$x_2(a) = 1 + f_1''(a) + ... + f_r''{}_{-1}a^{r-1} + a^r$ and

$m(a) = 1 + f_1'''(a) + ... + f_t'''{}_{-1}a^{t-1} + a^t$

where $s < r < t$ then,

$m(a) = 1 + f_1'''(a) + ... + f_t'''{}_{-1}a^{t-1} + a^t$

$x_1(a)+2x_2(a)+2um(a) = (1+f_1'(a)+......+f_{s-1}'a^{s-1}+a^s)+(2+2f_1''(a)+ .... + 2f_r''{}_{-1}a^{r-1} + 2a^r) + (2u + 2uf_1'''(a) + ... + 2uf_t'''{}_{-1}a^{t-1} + 2ua^t)$

$= (3+2u) + (f_1' + 2f_1'' + 2uf_1''')a + ... + (f_{t-1}' + 2f_t''{}_{-1} + 2uf_t'''{}_{-1})a^{t-1} + ... + (f_t' + 2f_t'' + 2uf_t'''a^t) + (f_{t+1}' + 2f_t''{}_{+1})a^{t+1} + ... + (f_{r-1}' + 2f_r''{}_{-1})a^{r-1} + (f_r' + 2)a^r + f_{r+1}'a^{r+1} + ... + f_{s-1}'a^{s-1} + a^s$

$(x_1(a) + 2x_2(a) + 2um(a))^{rc} = (1+u)(1+a+....+a^{n-s-2}) + ua^{n-s-1} + \overline{f_{s-1}'}a^{n-s} + ... + \overline{f_{r+1}'}a^{n-r-2} + (\overline{f_r'+2})a^{n-r-1} + (\overline{f_{r-1}' + 2f_r''{}_{-1}})a^{n-r} + ... + (\overline{f_{t+1}' + 2f_t''{}_{-1}})a^{n-t-2} + (\overline{f_t' + 2f_t'' + 2u})a^{n-t-1} + (\overline{f_{t-1}' + 2f_t''{}_{-1} + 2uf_t'''{}_{-1}})a^{n-t} + ... + (\overline{f_1' + 2f_1'' + 2uf_1'''})a^{n-2} + (\overline{3+2u})a^{n-1}$

$= (1+u)(1+a+....+a^{n-s-2}) + ua^{n-s-1} + \overline{f_{s-1}'}a^{n-s} + ... + \overline{f_{r+1}'}a^{n-r-2} + \overline{f_r'}a^{n-r-1} + 2a^{n-r-1} + \overline{f_{r-1}'}a^{n-r} + 2f_r''{}_{-1}a^{n-r} + ... + \overline{f_{t+1}'}a^{n-t-2} + 2f_t''{}_{+1}a^{n-t-2} + \overline{f_t'}a^{n-t-1} + c + 2ua^{n-t-1} + \overline{f_{t-1}'}a^{n-t} + 2f_t''{}_{-1}a^{n-t} + 2f_t'''{}_{-1}a^{n-t} + ... + \overline{f_1'}a^{n-2} + 2f''a^{n-2} + 2uf'''a^{n-2} + (2+3u)a^{n-1}$

$= (1+u)(1+a+.......+a^{n-s-2}) + ua^{n-s-1} + \overline{f_{s-1}'}a^{n-s} + ... + \overline{f_{r+1}'}a^{n-r-2} + \overline{f_r'}a^{n-r-1} + \overline{f_{r-1}'}a^{n-r} + \overline{f_{t+1}'}a^{n-t-2} + \overline{f_t'}a^{n-t-1} + \overline{f_{t-1}'}a^{n-t} + ... + \overline{f_1'}a^{n-2} + ua^{n-1} + 2a^{n-r-1} + 2f_r''{}_{-1}a^{n-r} + ... + 2f_t''{}_{+1}a^{n-t-2} + 2f_t''a^{n-t-1} + 2f_t''{}_{-1}a^{n-t} + ... + 2f_1''a^{n-2} + 2a^{n-1} + 2ua^{n-t-1} + 2uf_t'''{}_{-1}a^{n-t} + ... + 2uf_1'''a^{n-2} + 2ua^{n-1}$

$= (1+u)(1+a+.......+a^{n-s-2}) + ua^{n-s-1} + \overline{f_{s-1}'}a^{n-s} + ... + \overline{f_1'}a^{n-s} + ua^{n-1} + 3.2a^{n-r-1} + 3.2f_r''{}_{-1}a^{n-r} + ... + 3.2f_1''a^{n-2} + 3.2a^{n-1} + 3.2ua^{n-t-1} + 3.2f_t'''{}_1a^{n-t} + ... + 3.2uf_1'''a^{n-2} + 3.2ua^{n-1}$

Since C is linear code we must have,

$(x_1(a) + 2x_2(a) + 2m(a))^{rc} + 3(1+u)\frac{1-a^n}{1-a}\epsilon C$

That implies that,

$(x_1(a) + 2x_2(a) + 2m(a))^{rc}3(1+u)(\frac{(1-a^n)}{(1-a)}) = 3a^{n-s-1} + (\overline{f_{s-1}'} + 3(1+u))a^{n-s} + ... + (\overline{f_1'} + 3(1+u))a^n{-}^2 + 3a^{n-1} + 3.2a^{n-r-1}(1 + f_r''{}_{-1}a + f_1''a^{r-1} + a^r) + 3.2ua^{n-t-1}(1 + f_t'''{}_{-1}a + ... + f_1'''a^{t-1} + a^t)$

$= 3a^{n-s-1} + 3f_{s-1}'a^{n-s} + ... + 3f_1'a^{n-2} + 3a^{n-1} + 3.2a^{n-r-1}(1 + f_r''{}_{-1}a + ... + f_1''a^{r-1} + a^r) + 3.2ua^{n-t-1}(1 + f_t'''{}_{-1}a + ... + f_1'''a^{t-1} + a^t)$

$= 3a^{n-s-1}(1 + f_{s-1}'a + ... + f_1'a^{s-1} + a^s) + 3.2a^{n-r-1}(1 + f_r''{}_{-1}a + ... + f_1''a^{r-1} + a^r) + 3.2ua^{n-t-1}(1 + f_t'''{}_{-1}a + ... + f_1'''a^{t-1} + a^t)$

$= 3a^{n-s-}1x_1^*(a) + 3.2a^{n-r-}1x_2^*(a) + 3.2ua^{n-t-1}m^*(a)$

8

$= 3a^{n-s-1}(x_1^*(a) + 2a^{s-r}x_2^*(a) + 2ua^{s-t}m^*(a))$
$x_1^*(a) + 2a^{s-r}x_2^*(a) + 2ua^{s-t}m^*(a)\epsilon C.$
So, we have,
$x_1^*(a) + 2a^{s-r}x_2^*(a) + 2ua^{s-t}m^*(a) = x_1(a) + 2x_2(a) + 2um(a)x(a)).$
It is easy to see that, $x(a) = 1, (1 + u), 1 + 2u$ or $1 + 3u$.
$x_1(a) = x_1^*(a)$, i.e, $x_1(a)$ is self reciprocal.
Also we have $a^i x_2^*(a) = x_2(a)$ and $a^j m^*(a) = m(a)$ or $x_2(a)|(2a^j m^*(a) + 2m(a))$. where $i = s - r, j = s - t$
On the other hand, let $c(x)\epsilon C$, then
$c(a) = x_1(a) + 2x_2(a) + 2um(a)x(a)$
Since, $x_2(a)$ is self reciprocal. and also,
$a^i x_2^*(a) = x_2(a)$ and $a^j m*(a) = m(a)$ or $x_2(a)|(2a^j m^*(a)+2m(a))$ where $i = degx_2(a) - degx_1(a)$ and $j = degx_2(a) - degm(a)$ then we write,
$c^*(a)(x_1(a) + 2x_2(a) + 2um(a))x(a))^*$
$= (x_1^*(a) + 2a^i x_2^*(a) + 2ua^j m^*(a))x^*(a)$
$= (x_1(a) + 2x_2(a) + 2um(a))x^*(a)$
$= (x_1(a) + 2x_2(a) + 2um(a))c.x^*(a)$
where $c = 1, 1 + u, 1 + 2u$ or 1+3u therefore $c^*(a)\epsilon C$
Since $3(1 + u)(\frac{(1-a^n)}{(1-a)})\epsilon C$ we have,
$3(1 + u)\frac{(1-a^n)}{(1-a)} = 3(1 + u) + 3(1 + u)a + ... + 3(1 + u)a^{n-1}\epsilon C$
Let $c(a) = c_0 + c_1(a) + ... + c_m a^m \epsilon C$. As C is a cyclic code of length n, we have,
$a^{n-m-1}c(a) = c_0 + a^{n-m-1} + c_1 a^{n-m} + ... + c_m a^{n-1}\epsilon C$
whence,
$3(1+u) + 3(1+u)a + ... + 3(1+u)a^{n-m-2} + (c_0 + 3(1+u))a^{n-m-1} + (c_1 + 3(1 + u))a^n{}^{-m} +....... + (c_m + 3(1 + u))a^{n-1}$
$= 3(1+u)+3(1+u)a+.........+3(1+u)a^{n-m-2}+3\overline{c_0}a^{n-m-1}+3\overline{c_1}a^{n-m}+ ...... + 3\overline{c_m}a^{n-1}$
$= 3((1 + u) + (1 + u)a + ... + (1 + u)a^{n-m-2} + \overline{c_0}a^{n-m-1} + \overline{c_1}a^{n-m} + ... + \overline{c_m}a^{n-1}\epsilon C$
that implies,
$c^*(a)^{rc}\epsilon C$, therefore $(c(a)^{rc})^* = c(a)^{rc}\epsilon C$
$c^*(a)^{rc} = c(a)^{rc}\epsilon C.$

Theorem 4
Let $C = x_1(a) + 2x_2(a) + 2um(a), ux_3(a) + 2ux_4(a)$ be a cyclic code of even length n over R with $x_2(a)|x_1(a)|(a^{n-1})$ and $x - 4(a)|x_3(a)|(a^{n-1})$ in $\frac{R[a]}{a^{n-1}}$. Then C is a reverse complement if and only if,
(i) $3(1 + u)(\frac{(1-a^n)}{(1-a)})\epsilon C$ and $x_1(a)$ and $x_3(a)$ are self reciprocal.
(ii) $a^i x_2^*(a) = x_2(a)$ and

9

$x_4(a)|(2a^j m^*(a) + 2m(a))$ or
$x_4(a)|(2a^j m^*(a) + 2m(a) + 2x_2(a))$
where $i = degx_1(a) - degx_2(a)$, $j = degx_1(a) - degm(a)$.

Proof

Let $C = x_1(a)+2x_2(a)+2um(a), ux_3(a)+2ux_4(a)$ with $x_2(a)|x_1(a)|(a^{n-1})$ and $x - 4(a)|x_3(a)|(a^{n-1})$ in $\frac{R[a]}{a^{n-1}}$. the WCC of it should also be in C.

$i.e, \overline{3(0, 0, 0, ..., 0} = 3(1 + u, 1 + u, ..., 1 + u) = 3(1 + u)\frac{1-a^n}{1-a}\epsilon C$

$x_1^*(a) + 2a^{s-r}x_2^*(a) + 2ua^{s-t}m^*(a)\epsilon C$

$degx_1(a) = s, degx_2(a) = r, degm(a) = t$

So we have,

$x_1^*(a) + 2a^{s-r}x_2^*(a) + 2ua^{s-t}m^*(a) = x_1(a) + 2x_2(a) + 2um(a)x(a) + u(x_3(a) + 2x_4(a))b(a)$ (1)

Here $x(a) = 1, 1 + u, 1 + 2u$, or $1 + 3u$

$\therefore x_1(a) = x_1^*(a), x_1(a)$ is self reciprocal.

Also we have,

$a^i x_2^*(a) = x_2(a)$

Now suppose,

$ux_3(a) = uf_1'(a) + ... + uf_{k-1}'a^{k-1} + ua^k$ (2)

$2ux_4(a) = 2u + 2uf_1''(a) + ... + 2uf_l''{}_{-1}a^{l-1} + 2ua^l$ (3)

where $k < l$ then,

$ux_3(a)+2ux_4(a) = (u+2u)+(uf_1'+2uh_1'')a+.......+(uf_{l-1}'+2uf_l''{}_{-1})a^{l-1}+ (uf_l' + 2u)a^l + uf_{l+1}'a^{l+1} + ... + uf_{k-1}'a^{k-1} + ua^k$

$(ux_3(a) + 2ux_4(a))^{rc} = (1 + u)(1 + a + ... + a^{n-k-2}) + a^{n-k-1} + \overline{uf_{k-1}'}a^{n-k}+........+\overline{uf_{l+1}'}a^{n-l-2}+(\overline{uf_l' + 2u})a^{n-l-1}+(\overline{uf_{l-1}' + 2uf_l''{}_{-1}})a^{n-1}+ .... + (\overline{uf_1' + 2uf_1''})a^{n-2} + (1 + 2u)a^{n-1}$

$= (1+u)(1+a+.......+a^{n-k-2})+a^{n-k-1}+\overline{uf_{k-1}'}a^{n-k}+........+\overline{uf_{l+1}'}a^{n-l-2}+ \overline{uf_1'}a^{n-l-1}+2ua^{n-l-1}+\overline{uf_{l-1}'}a^{n-l}+2uf_l''{}_{-1}a^{n-l}+.........+\overline{uf_l'}a^{n-2}+2uf_1''a^{n-2}+ 2ua^{n-1} + a^{n-1}$

$= (1+u)(1+a+.......+a^{n-k-2})+a^{n-k-1}+\overline{uf_{k-1}'}a^{n-k}+........+\overline{uf_{l+1}'}a^{n-l-2}+ \overline{uf_l'}a^{n-l-1}+\overline{uf_{l-1}'}a^{n-1}+.......+\overline{uf_1'}a^{n-2}+a^{n-1}+3.2ua^{n-l-1}+3.2uf_l''{}_{-1}a^{n-l}+ ...... + 3.2uf_1''a^{n-2} + 3.2ua^{n-1}$

Since C is a linear code we must have,

$(ux_3(a) + 2ux_4(a))^{rc} + 3(1 + u)\frac{1-a^n}{1-a}\epsilon C$

$(ux_3(a) + 2ux_4(a))^{rc} + 3(1 + u)(\frac{(1-a^n)}{(1-a)})$

$= 3ua^{n-k-1} + \overline{uf_{k-}'1} + 3(1 + u)a^{n-k} + ... + \overline{uf_1'} + 3(1 + u)a^{n-2} + 3ua^{n-1} + 3.2a^{n-l-1}(u + uf_l''{}_{-1}a + ... + uf_1''a^{l-1} + ua^l$

$= 3ua^{n-k-1} + 3uf_{k-1}'a^{n-k} + ... + 3uf_1'a^{n-2} + 3ua^{n-1} + 3a^{n-l-1}(2u + 2uf_l''{}_{-1}a + ... + 2uf_1''a^{l-1} + 2ua^l)$

$= 3a^{n-k-1}(u+uf_{k-1}'a+.........+uf_1'a^{k-1}+ua^k)+3a^{n-l-1}(2u+2uf_l''{}_{-1}a+ .... + 2uf_1''a^{l-1} + 2ua^l)$

10

$= 3a^{n-k-1}ux_3^*(a) + 3.2ua^{n-l-1}x_4^*(a)$
$= 3a^{n-k-1}(ux_3^*(a) + a^{k-l}2um^*(a))\epsilon C$
Therefore, $x - 3(a) = x_3^*(a)$ by equation (1) we have,
$x_4(a)|(2a^j m^*(a) + 2m(a))$ (or)
$x_4(a)|(2a^j m^*(a) + 2m(a) + 2x_2(a))$
where $j = degx_1(a) - degm(a)$
Conversely, Let $c(a)\epsilon C$ then,
$c(a) = x_1(a) + 2x_2(a) + 2um(a)g(a) + (ux_3(a) + 2ux_4(a))h(a)$.
Since $x_1(a)$ and $x_3(a)$ are self reciprocal,
$a^i x_2^*(a) = x_2(a)$ and $x_4(a)|(2a^j m^*(a) + 2m(a)$
(or) $x_4(a)|(2a^j m^*(a) + 2m(a) + 2x_2(a))$
Where $i - degx_1(a) - degx_2(a)$
$j = degx_1(a) - degm(a)$ then we write,
$c^*(a) = ((x_1(a) + 2x_2(a) + 2um(a))g(a))^* + ((ux_3(a) + 2ux_4(a))h(a))^*$
$= x_1^*(a) + 2a^i x_2^*(a) + 2ua^j m^*(a))g^*(a) + (ux_3^*(a) + 2ua^k x_4^*(a))h^*(a)$
$= x_1(a) + 2x_2(a) + 2um(a)g^*(a) + (2ua^j m^*(a) + 2um(a)g^*(a)) + (ux_3(a) + 2ua^k x_4^*(a))h^*(a)$
$= (x_1(a) + 2x_2(a) + 2um(a))g^*(a) + ux_4(a)$ as, $x_4(a)|x_3(a)$
where $c = 1, 1 + u, 1 + 2u$ or $1 + 3u$
$\therefore c^*(a)\epsilon C$
As, $3(1 + u)(\frac{(1-a^n)}{(1-a)})\epsilon C$ we have,
$c(a) = c_0 + c_1 a + ... + c_m a^m \epsilon C.$
Since C is a cyclic code of length n we have,
$a^{n-m-1}c(a) = c_0 a^{n-m-1} + c_1(a)^{n-m} + ... + c_m a^{n-1}\epsilon C.$
Hence, $3(1 + u) + 3(1 + u)a + ... + 3(1 + u)a^{n-m-2} + (c_0 + 3(1 + u))a^{n-m-1} + (c_1 + 3(1 + u))a^{n-m} + ... + (c_m + 3(1 + u))a^{n-1}$
$= 3(1+u)+3(1+u)a+........+3(1+u)a^{n-m-2}+3\overline{c_0}a^{n-m-1}+3\overline{c_1}a^{n-m}+ .... + 3\overline{c_m}a^{n-1}$
$= 3((1 + u) + (1 + u)a + ... + (1 + u)a^{n-m-2} + \overline{c_0}a^{n-m-1} + \overline{c_1}a^{n-m} + ... + \overline{c_m}a^{n-1})\epsilon C$
This implies, $c^*(a)^{rc}\epsilon C$
$\therefore (c^*(a)^{rc})^* = c(a)^{rc}\epsilon C.$

The above theorem is illustrated the following example,
$(x^6 - 1) = (x - 1)(x + 1)(x^2 + x + 1)(x^2 - x - 1)$
Let $C = x_1(a) + 2x_2(a) + 2um(a), ux_3(a) + 2ux_4(a)$. Where $x_1(a) = x_2(a) = h_2$ $x_3(a) = h_1, x_4(a) = 1$ and $m(a) = 0$ we check that $x_1(a)$ and $x_3(a)$ are self reciprocal. $a^i x_2^*(a) = x_2(a)$ and $x_4(a)|(2a^j m^*(a) + 2m(a))$ the hamming distance 6 and a DNA code of length 6 are given in Table - 2.

11

A DNA code length 6 obtained from

TTTTTT	TATTAT	TCTTCT	TGTTGT
ATTATT	AATAAT	ACTACT	AGTAGT
CTTCTT	CATCAT	CCTCCT	CGTCGT
GTTGTT	GATGAT	GCTGCT	GGTGGT
TTATTA	TAATAA	TCATCA	TGATGA
ATAATA	AAAAAA	ACAACA	AGAAGA
CTACTA	CAACAA	CCACCA	CGACGA
GTAGTA	GAAGAA	GCAGCA	GGAGGA
TTCTTC	TACTAC	TCCTCC	TGCTGC
ATCATC	AACAAC	ACCACC	AGCAGC
CTCCTC	CACCAC	CCCCCC	CGCCGC
GTCGTC	GACGAC	GCCGCC	GGCGGC
TTGTTG	TAGTAG	TCGTCG	TGGTGG
ATGATG	AAGAAG	ACGACG	AGGAGG
CTGCTG	CAGCAG	CCGCCG	CGGCGG
GTGGTG	GAGGAG	GCGGCG	GGGGGG

Table-2

Conclusion

We have investigated some properties of generator polynomial of reversible and reversible complement skew cyclic codes with even length over the ring $F_2 + UF_2 + VF_2$ using the mathematical structure of DNA . For future study the algebraic structure of cyclic codes of even length and their relation to DNA codes is still an open problem.

Acknowledgment

The authors are very grateful to the anonymous referees for their comments and suggestions which helped significantly in improving the paper.

Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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