Global Analysis of an SEIQV Epidemic Model For COVID-19 With Vaccination Strategy

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ABSTRACT:

In this paper we study the dynamics of COVID-19 using an SEIQV epidemic model with asymptotically homogeneous transmission function is consider. The basic reproduction number R_0 is found with help of Next Generation Matrix. Global stability of Disease-free equilibrium is obtained by Lyapunov function if $R_0 < 1$, then Disease free equilibrium is globally asymptotically stable . And Endemic equilibrium is globally asymptotically stable if $R_0 > 1$.

Keywords: COVID-19, Homogenous Transmission Function, SEIQV epidemic model, Lyapunov function, Routh-Hurwitz Criterion.

1. INTRODUCTION

Mathematical models are useful to the behavior of an infection when it enters a community and investigate under which condition it will be wiped out or continued.

The novel Coronavirus disease (COVID-19) was first confirmed in the China city of Wuhan , late December, 2019. The coronavirus disease caused by the Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) presents clinical features which are similar to the disease caused by other coronaviruses, Severe Acute Respiratory Syndrome (SARS) and Middle East Respiratory Syndrome such as lower respiratory illness with fever, dry cough, myalgia, shortness of breath etc. The incubation period of COVID-19 is between 2-14 days with symptoms averagely between 5-7 days. Its basic reproduction number is averaged 2.2[16] and even more ranging from 1.4-6.5 in [5]. The Coronavirus

disease has to be control by effective lockdown ,social distancing, wearing mask ,quarantine and vaccination.

Since the outbreak, many mathematical have appeared in an attempt to assess the dynamic mathematics models aimed at estimating of the basic reproduction number R_0 [18,6,9,1] and Nezihal Gokbulut et. all [4] recently discovered an SVI mathematical model. Vaccine for overall human can reduced the risk of future COVID-19.

In this paper we aim to study the dynamical behavior of an corona virus which has some effective vaccine and can recovered either by naturally or by some proper treatments. We present an SEIQV(Susceptible-Exposed-Infected-Recovered) epidemic model with Homogenous Transmission function (i.e. for large population sizes the transmission rate will be approximately proportional to the fraction of infectives in the total population) and Vaccination strategy which prescribe inhibitory measure such as personal hygiene, wearing of face mask, public gathering ban, and increased medical facilities etc. as potent means of solving down the spread of COVID-19.

2. Mathematical Model

The model we analyze in this paper is consider of the following nonlinear ordinary differential equation:

$$\frac{dS}{dt} = b - \frac{KSI}{1+\alpha S+\beta I} - (\omega+\mu+q_3)S$$

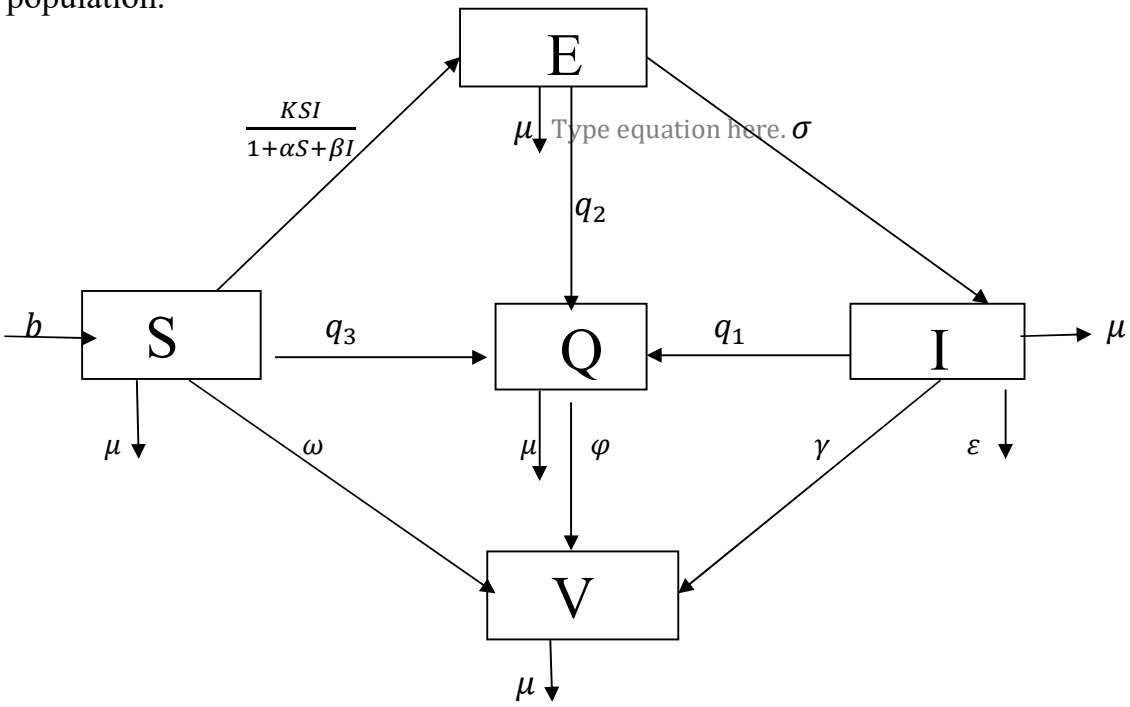
$$\frac{dE}{dt} = \frac{KSI}{1+\alpha S+\beta I} - (\sigma+\mu+q_2)E$$

$$\frac{dI}{dt} = \sigma E - (\mu+\varepsilon+\gamma+q_1)I$$

$$\frac{dQ}{dt} = q_3 S + q_2 E + q_1 I - (\mu-\phi)Q$$

$$\frac{dV}{dt} = \omega S + \phi Q + \gamma I - \mu V \qquad .....(2.1)$$

Where S(t), E(t), I(t), and Q(t) denote the number of susceptible individuals, exposed individuals but not yet infectious, infectious individual, quarantined – treated individuals at time t, respectively, and V(t) represents the total population of recovered individuals and vaccinated – treated individuals at time t. And b is the recruitment rate of the population ,k is the disease transmission coefficient ,μ is the natural death rate of the population, ε is the death rate for disease of infectious individuals , and $\gamma, \sigma, \omega, \varphi, q_1, q_2, q_3$ are the state transition rates. α *and* $\beta$are the parameter measures the psychological or inhibitory effect and sociological respectively. The transmission rate $\frac{KSI}{1+\alpha S+\beta I}$ is a saturation effect accounting for the fact that the number of contact an individual reaches some maximal value due to social distribution of the population.



Let N(t) represents that total population at time *t*, which satisfied.

$$N(t) = S(t) + E(t) + I(t) + Q(t) + V(t)$$

$$\frac{dN}{dt} = b - \mu N - \varepsilon I$$

$$\frac{dN}{dt} \le b - \mu N$$

$$\lim_{n\to\infty}(S+E+I+Q+V) \le \frac{b}{\mu}$$

So, the feasible region for system

$$\Omega = \left\{ (S,E,I,Q,V) \in R^5_+ \mid 0 \le S+E+I+Q+V \le \frac{b}{\mu}, S>0, E \ge 0, I \ge 0, Q>0, V>0 \right\}$$

The set Ω is positively Invariant with respect to system (2.1)

2.1 Equilibrium of the System

Solution of the system 2.1

$$b - \frac{KSI}{1+\alpha S+\beta I} - (\omega+\mu+q_3)S = 0$$

$$\frac{KSI}{1+\alpha S+\beta I} - (\sigma+\mu+q_2)E = 0$$

$$\sigma E - (\mu+\varepsilon+\gamma+q_1)I = 0$$

$$q_3 S + q_2 E + q_1 I - (\mu+\phi)Q = 0$$

$$\omega S + \phi Q + \gamma I - \mu V = 0 \qquad (2.2)$$

For system (2.2) always has a Disease Free Equilibrium $P_0(S_0, o, o, Q_0, V_0)$

Where, $S_0 = \frac{b}{\omega+\mu+q_3}, Q_0 = \frac{q_3 S_0}{\mu+\phi}, V_0 = \frac{\omega S_0 + \phi Q_0}{\mu}$

And for system 2.2 has a unique positive solution $P_*(S^*, E^*, I^*, Q^*, V^*)$

$$\boxed{S^* = \frac{(1+\beta I)(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)}{\sigma K - \alpha(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)}}$$

$$Q^* = \frac{q_3 S + q_0 E + q_1 I}{\mu+\phi}, E^* = \frac{(\mu+\varepsilon+\gamma+q_1)}{\sigma}$$

$$V^* = \frac{\omega S + \phi Q + \gamma I}{\mu}, \quad I^* = \frac{\sigma(\omega+\mu+q_3+\alpha b)(R_0-1)}{\sigma\beta(\omega+\mu+q_3) + \sigma K - \alpha(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)}$$

From the value of I^* it is obvious that all the values of (S^*, E^*, Q^*, V^*) are positive if $R_0 > 1$.

2.2 Reproduction Number (R_0)

The Reproduction number, measure the probability of an infectious disease spreading through a population or becoming extinct after sometimes. This threshold characteristics of R_0 help epidemiologist to make the following assumption: (a) If $R_0 < 1$, the infection will die-out with time and (b) If $R_0 > 1$, the disease will be endemic in the population. Next we find the Reproduction number R_0 of the system (2.1) by obtaining the Jacobian of the system and using the Next –Generation Matrix due to Driessche and watmough[14].

Let $X=(E,I)^T$ system (2.1) become $\frac{dX}{dt} = F(X) - V(X)$

$$F(X) = \begin{bmatrix} \frac{KSI}{1+\alpha S+\beta I} \\ 0 \end{bmatrix}, V(X) = \begin{bmatrix} (\sigma+\mu+q_2)E \\ -\sigma E + (\mu+\varepsilon+\gamma+q_1)I \end{bmatrix}$$

Jacobian of F(X) and V(X) at Disease Free Equilibrium P_0 $(S_0,0,0,Q_0,V_0)$

$$F = \begin{bmatrix} 0 & \frac{Kb}{\omega+\mu+q_3+\alpha b} \\ 0 & 0 \end{bmatrix}, V = \begin{bmatrix} (\sigma+\mu+q_2) & 0 \\ -\sigma & (\mu+\varepsilon+\gamma+q_1) \end{bmatrix}$$

The Reproduction number R_0 is the spectral radius of the Next-Generation Matrix derived from the exposed and infected class i.e. $R_0 = \rho(fV^{-1})$ which is

$$F.V^{-1} = \begin{bmatrix} 0 & \frac{Kb}{\omega+\mu+q_3+\alpha b} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma+\mu+q_2} & 0 \\ \frac{\sigma}{(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)} & \frac{1}{(r+\varepsilon+\gamma+q_1)} \end{bmatrix}$$

Hence the Reproduction number of system (2.1) is

$$R_0 = \rho(F.V^{-1}) = \frac{\sigma Kb}{(\omega+\mu+q_3+\alpha b)(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)}$$

3. Stability of Equilibriums

In this section we will discuss about the stability of Disease- free Equilibrium $P_0(S_0,0,0,Q_0,V_0)$ and Endemic Equilibrium $P_*(S^*,E^*,I^*,Q^*,V^*)$.

3.1 Disease-free -Equilibrium

Theorem 3.1.1: The system (2.1) is locally asymptotically stable to Disease Free Equilibrium point P_0, If $R_0 < 1$ and unstable if $R_0 > 1$.

Proof: The Jacobian of system (2.1) at Disease-Free equilibrium P_0 is

$$J(P_0)=\begin{bmatrix} -(\omega+\mu+q_3+\phi) & 0 & \dfrac{-Kb}{\omega+\mu+q_3+\alpha b} & 0 & 0 \\ 0 & -(\sigma+\mu+q_2) & \dfrac{Kb}{\omega+\mu+q_3+\alpha b} & 0 & 0 \\ 0 & \sigma & -(\mu+\varepsilon+\gamma+q_1) & 0 & 0 \\ q_3 & q_2 & q_1 & -(\mu+\phi) & 0 \\ \omega & 0 & \gamma & \phi & -r \end{bmatrix}$$

Characteristics equation of Jacobian and it is obvious that $J(P_0)$ has always three negative Eigen values

$$\lambda_1=-(\omega+\mu+q_3),\lambda_2=-(\mu+\phi),\lambda_3=-\mu$$

And the other Eigen values of $J(P_0)$ are determined by the equation.

$$\lambda^2+(2\mu+\sigma+\varepsilon+\gamma+q_1+q_2)\lambda+(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)(1-R_0)=0 \qquad .....(3.1)$$

If $R_0>1$, then equation (3.1) has a positive root and a negative root. The DFE P_0 is unstable saddle point.

When $R_0 \le 1$. then both roots are negative therefore, the Disease Free Equilibrium is stable.

Theorem 3.1.2: The system (2.1) is Globally asymptotically stable if $R_0<1$ at Disease Free Equilibrium and disease will be die-out.

Proof: We Construct the Lyapunov function

$$L(E,I)=\sigma E+(\sigma+\mu+q_2)I$$

The derivation L^* with respect to *it gives*

$$L' = \sigma E' + (\sigma + \mu + q_2) I'$$

$$= \sigma\left(\frac{KSI}{1+\alpha S+\beta I} - (\sigma+\mu+q_2)E\right) + (\sigma+\mu+q_2)\left(\sigma E - (\mu+\varepsilon+\gamma+q_1)I\right)$$

$$\frac{dL}{dt} \le (\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1)[R_0 - 1]I \le 0$$

If $R_0 \le 1$, the largest compact Invariant set is the singleton set $P_0(S_0,E_0,I_0,Q_0,V_0)$ in $\{(S,E,I,Q,V): \frac{dL}{dt}=0\}$. Thus by the Lasalle- Invariance Principle, the disease-free equilibrium is Globally asymptotically stable in **Ω**.

Here $\frac{dL}{dt} = 0$ if only $I = 0$ or $P_0 = (S_0, E_0, I_0, Q_0, V_0)$

3.2 Stability for Endemic Equilibrium

Theorem 3.2.1: If $R_0 > 1$, the endemic equilibrium P* is locally asymptotically stable.

Proof: The Jacobian matrix of system (2.1) at P* is

$$J(P^*) = \begin{bmatrix} -(\omega+\mu+q_3)\frac{-(1+\beta I^*)KI^*}{(1+\alpha S+\beta I^*)^2} & 0 & \frac{-(1+\sigma S^*)KS^*}{(1+\alpha S+\beta I^*)^2} & 0 & 0 \\ \frac{(1+\beta I^*)KI^*}{(1+\alpha S+\beta I^*)2} & -(\sigma+\mu+q_2) & \frac{(1+\alpha S^*)KS^*}{(1+\alpha S^*+\beta I^*)^2} & 0 & 0 \\ 0 & \sigma & -(\mu+\varepsilon+\gamma+q_1) & 0 & 0 \\ q_3 & q_2 & q_1 & -(\mu+\phi) & 0 \\ \omega & 0 & \gamma & \phi & -\mu \end{bmatrix}$$

$$Z_1 = \frac{(1+\beta I^*)KI^*}{(1+2S^*+\beta I^*)^2}, Z_2 = \frac{(1+\alpha S^*)KS^*}{(1+\alpha S+\beta I^*)^2}$$

It is obvious two roots are negative $\lambda_1 = -\mu, \lambda_2 = -(\mu+\phi)$

And characteristics equation of P^* is

$$\lambda^3 + \lambda^2(3\mu+\omega+\varepsilon+\gamma+\sigma+q_1+q_2+q_3) + \lambda[(2\mu+\sigma+\varepsilon+\gamma+q_1+q_2)$$
$$(\omega+\mu+q_3+Z_1) + (\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1) - \sigma Z_2]$$
$$+(\omega+\mu+q_3+Z_1)(\sigma+\mu+q_2)(\mu+\varepsilon+\gamma+q_1) - \sigma Z_2(\omega+\mu+q_3+Z_1) + \sigma Z_1 Z_2 = 0$$

then we only need to consider the roots of

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0, \text{where}$$

$$A = 3\mu + \omega + \varepsilon + \gamma + \sigma + q_1 + q_2 + q_3 > 0$$

$$B = (2\mu + \sigma + \varepsilon + \gamma + q_1 + q_2)(\omega + \mu + q_3 + \frac{(1+\beta I^*)KI^*}{(1+\alpha S^* + \beta I^*)^2}) +$$

$$(\sigma + \mu + q_2)(\mu + \varepsilon + \gamma + q_1) - \sigma\frac{(1+\alpha S^*)KS^*}{(1+\alpha S^* + \beta I^*)^2} > 0$$

$$C = (\omega + \mu + q_3 + \frac{(1+\beta I^*)KI^*}{(1+\alpha S^* + \beta I^*)^2})\left[(\sigma + \mu + q_2)(\mu + \varepsilon + \gamma + q_1) - \sigma\frac{(1+\alpha S^*)KS^*}{(1+\alpha S^* + \beta I^*)^2}\right] +$$

$$\sigma\frac{(1+\beta I^*)KI^*}{(1+\alpha S^* + \beta I^*)^2}\frac{(1+\alpha S^*)KS^*}{(1+\alpha S^* + \beta I^*)^2} > 0$$

$$\text{B} > 0, \text{C} > 0 \quad If\ (\sigma + \mu + q_2)(\mu + \varepsilon + \gamma + q_1) < \sigma\frac{(1+\alpha S^*)KS^*}{(1+\alpha S^* + \beta I^*)^2}$$

By direct calculation on

$AB - C > 0,$ therefor by theorem of Routh-Hurwitz it follows that the endemic equilibrium P^* is locally asymptotically stable. This completes the proof.

Definition 3.2: The system (2.1) is said to be uniformly persistent in Ω, if there exists a constant $c > 0$ such that any solution $(S(t), E(t), I(t), Q(t), V(t))$ of system (1.1) with initial value $(S(0), E(0), I(0), Q(0), V(0)) \in \text{int}\,\Omega$ satisfies.

$$\min\left\{\liminf_{t\to\infty} S(t), \liminf_{t\to\infty} E(t), \liminf_{t\to\infty} I(t), \liminf_{t\to\infty} Q(t), \liminf_{t\to\infty} V(t)\right\} \geq c.$$

Similar to [3], we can get

Theorem 3.3: System (2.1) is uniformly persistent in Ω if and only if $R_0 > 1.$

Remarks 32. The uniform persistence of system (2.1) in the bounded set Ω is equivalent to the existence of a compact $K \subset \Omega$ that is absorbing for (2.1) (see [5]).

Next we investigate the global stability of the endemic equilibrium of model (2.1). A geometrical approach developed in [12] (see also [11, 13, 3]) for

proving global stability will be used in our discussion. Such method is based on the use of a high-order generalization of the Bendixons criterion which precludes the existence of non-constant periodic solutions (see[20]).

Now we briefly outline a general mathematical framework developed in [12] for proving global stability. Consider the autonomous dynamical system.

$$\frac{dx}{dt} = f(x)$$

where $x \to f(x) \in R^n$ is a C^1 function about x in $\Omega_1 \subset R^n$. Assume that the following hypothesis hold:

(H_1): There is a compact absorb set $K \subset \Omega_1$;

(H_2): Differential equation (*) has a unique equilibrium x_* in Ω_1.

Let $x \to P(x)(\binom{n}{2} \times \binom{n}{2})$ matrix-valued function that is C^1 for $x \in \Omega_1$. Assume that $P^{-1}(x)$ exists and is continuous for $x \in K$. A quantity q is defined as

$$q = \lim_{t\to\infty} \sup \sup_{x\in K} \frac{1}{t} \int_0^t \mu(B(x(s, x_0)))ds$$

where $B = P_f P^{-1} + P \frac{\partial f[2]}{ax} p^{-1}$, the matrix P_f is obtained by replacing each entry P_{ij} of P by its derivative in the direction of f. The quantity $\mu(B)$ is the *Lazinski* measure of B with respect to a vector norm $|\cdot|$ in $R^N, N = \binom{n}{2}$, defined by $\mu(B) = \lim_{t\to 0} + \frac{|I + hB| - 1}{h}$.

The following global stable result is proved in Theorem 3.5 or [12].

Lemma 3.4: Suppose that Ω_1 is simply connected and that assumption (H_1)–(H_2) hold, then the unique equilibrium x_* is globally stable in Ω_1 if $q < 0$.

Now we apply the theory developed in [12], in particular Lemma 3.4, to prove the global stability of E^*.

Theorem 3.2: If $R_0 > 1$, the endemic equilibrium P* is Globally Asymptotically

Proof: Firstly we consider sub-system of (2.1)

$$\frac{ds}{dt} = b - \frac{KSI}{1+\alpha S+\beta I} - (\omega+\mu+q_3)S \qquad \frac{dE*}{dt} = \frac{KSI}{1+\alpha S+\beta I} - (\sigma+\mu+q_2)E$$

$$\frac{dI}{dt} = \sigma E - (\mu+\varepsilon+\gamma+q_1)I \qquad \text{.....(3.2)}$$

The Jacobian matrix of system 3.2 is

$$J(P^*) = \begin{bmatrix} -(\omega+\mu+q_3) - \frac{(1+\beta I*)KI*}{(1+\alpha S+\beta I)^2} & 0 & \frac{-KS*(1+\alpha S*)}{(1+\alpha S+\beta I*)^2} \\ \frac{(1+\beta I*)KI*}{(1+\alpha S+\beta I*)^2} & -(\sigma+\mu+q_2) & \frac{KS(1+\alpha S*)}{(1+\alpha S*+\beta I)^2} \\ 0 & \sigma & -(\mu+\varepsilon+\gamma+q_1) \end{bmatrix}$$

And its 2nd additive compound matrix is

$$J^{[2]} = \begin{bmatrix} \frac{-(1+\beta I*)KI*}{(1+\alpha S+\beta I*)^2} - m & \frac{KS*(1+\alpha S*)}{(1+\alpha S*+\beta I*)^2} & \frac{KS^4(1+\alpha S*)}{(1+\alpha S*+\beta I*)^2} \\ \sigma & -\frac{(1+\beta I*)KI*}{(1+\alpha S+\beta I*)^2} - n & 0 \\ 0 & \frac{(1+\beta I*)KI*}{(1+\alpha S+\mathrm{B}I*)^2} & -k \end{bmatrix}$$

where, $m = 2\mu+\omega+\sigma+q_2+q_3$

$n = 2\mu+\omega+\varepsilon+\gamma+q_1+q_3$

$k = 2\mu+\sigma+\varepsilon+\gamma+q_1+q_2$

Choose the function $P(x) = P(S,E,I) = diag\left(1, \frac{E}{I}, \frac{E}{I}\right)$,

It follows that

$$P_f P^{-1} = \text{diag}\left(0, \frac{E'}{E} - \frac{I'}{I}, \frac{E'}{E} - \frac{I'}{I}\right)$$

$$PJ^{[2]}, P^{-1} \Rightarrow \begin{bmatrix} \frac{-KI*(1+\beta I*)}{(1+\alpha S*+\beta I*)^2} - m & \frac{KS*I(1+\alpha S*)}{E(1+\alpha S^*+\beta I*)^2} & \frac{KS*I(1+\alpha S*)}{E(1+\alpha S*+\beta I*)^2} \\ \sigma\frac{E}{I} & \frac{-KI*(1+\beta I*)}{(1+\sigma S*+\beta I*)^2} - n & 0 \\ 0 & \frac{KI^*(1+\beta I*)}{(1+\alpha S^*+\beta I^*)^2} & -k \end{bmatrix}$$

The matrix $B = P_f P^{-1} + PJ^{[2]}P^{-1}$ can be written in matrix form

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$B_{11} = \frac{-KI^*(1+\beta I^*)}{(1+\alpha S^*+\beta I^*)^2} - m$$

$$B_{12} = \left(\frac{KS^*I(1+\alpha S^*)}{E(1+\alpha S^*+\beta I^*)^2}, \frac{KS^*I(1+\alpha S^*)}{E(1+\alpha S^*+\beta I^*)^2} \right)$$

$$B_{21} = \left(\frac{\sigma E}{I}, 0 \right)$$

$$B_{22} = \begin{pmatrix} \frac{E'}{E} - \frac{I'}{I} - \frac{KI^*(1+\beta I^*)}{(1+\alpha S^*+\beta I^*)^2} - n & 0 \\ \frac{KI^*(1+\beta I^*)}{(1+\alpha S^*+\beta I^*)^2} & \frac{E'}{E} - \frac{I'}{I} - k \end{pmatrix}$$

Let (μ, v, ω) be vector in $\square^3$ its norm 11.11 is defined as

$$\| \mu, v, \omega \| = \max \{ | \mu |, | v | - | \omega | \}$$

Let $\mu(B)$ be the Lozinski measure with respect to this norm, then as described in []. We choose

$$\mu(B) \leq \sup \{g_1, g_2\}$$

where $g_1 = \mu_1(B_{11}) + | B_{12} |,$

$$g_2 = | B_{11} | + \mu_1(B_{22})$$

$| B_{12} |, | B_{21} |$ are matrix norms with respect to the l_1, vector norm μ_1, denotes the Lozinski measure with respect to this l_1 norm. then

$$\mu_1(B_{11}) = \frac{-KI(1+\beta I)}{(1+\alpha S+\beta I)^2} - m$$

$$|B_{21}| = \frac{\sigma E}{I}$$

$$|B_{12}| = \max\left\{\frac{KSI(1+\alpha S)}{E(1+\alpha S+\beta I)^2}, \frac{KSI(1+\alpha S)}{E(1+\alpha S+\beta I)^2}\right\} = \frac{KSI(1+\alpha S)}{E(1+\alpha S+\beta I)^2}$$

and $\mu_1(B_{22}) = \frac{E'}{E} - \frac{I'}{I} - \min\{n,k\}$ .... (3.3)

Therefore, we have

$$g_1 = \frac{-KI^*(1+\beta I^*)}{(1+\alpha S^*+\beta I*)^2} - m + \frac{KS^*I(1+\alpha S^*)}{E(1+\alpha S^*+\beta I^*)^2}$$

$$\leq \frac{KI^*(1+\beta I^*)}{(1+\alpha S^*+\beta I^*)^2} - (\mu+\omega+q_3) + \frac{E'}{E} \quad \text{....(3.4)}$$

$$g_2 = \frac{I'}{I} + (\mu+\varepsilon+\gamma+q_1) + \frac{E'}{E} - \frac{I'}{I} - \min\{n,k\}$$

$$\leq \frac{E'}{E} - \mu$$

$$\mu(B) \leq \sup\{g_1, g_2\} \leq \sup\left\{\frac{-KI(1+\beta I)}{(1+\alpha S+\beta I)^2} - \mu + \frac{E'}{E}, \frac{E'}{E} - \mu\right\}$$

$$\leq \frac{E'}{E} - \mu,$$

then $\frac{1}{t}\int_0^t \mu(B)ds \leq \frac{1}{t}\int_0^t\left(\frac{E'}{E} - \mu\right)ds = \frac{1}{t}\ln\frac{E(t)}{E(o)} - \mu$

which implies $q \leq \frac{-\mu}{2} < 0$. Then based on theorem 3.5 of [].

We know that equilibrium (S_*, E_*, I_*) is globally asymptotically stable.

Now consider the sub-system (2.1)

$$\left.\begin{aligned}\frac{dQ}{dt} &= q_3S + q_2E + q_1I - (\mu+\phi)Q\\ \frac{dv}{dt} &= \omega S + \phi Q + \gamma I - \mu V\end{aligned}\right\} \quad \text{....(3.5)}$$

and its limit system is based on 3.5 we get

$$Q(t) = e^{-(\mu+\theta)t}\left[Q(0) + q_3 S + q_2 E + q_1 I\int_0^t e^{(\mu+Q)S} ds\right]$$

$$V(t) = e^{-\mu t}\left[V(0) + \int_0^t e^{\mu S}(\omega S_* + \phi Q(s) + \gamma I_*) ds\right]$$

Which implies that

$$Q(t) \Rightarrow \frac{q_3 S_* + q_2 E_* + q_1 I_*}{\mu + Q} = Q_*, V(t) \Rightarrow \frac{\omega S_* + \phi Q_* + \gamma I_*}{\mu} = V_*, t \to \infty$$

Then we get that E* is globally asymptotical stable. The proof is completed.

4. Conclusion

This paper presents a mathematical study on the global dynamics of an SEIQV epidemic model for COVID -19 with homogenous transmission rate with vaccination strategy .In case of COVID-19 we quarantine susceptible ,exposed and Infected individual . It is rigorously established in Theorems 3.1.2 and 3.2.2 that the basic reproduction number is a sharp threshold parameter and completely determines the global dynamics of (2.1) in the feasible region. We conclude that in Increasing vaccination coverage decrease total symptomatic cases in each scenario. If R_0 <1 , there is the disease free equilibrium is globally asymptotically stable, that is the disease die- out and when R_0 > 1, it become endemic equilibrium is globally stable in **Ω** and disease will persist. The proof of the global stability of P^* when R_0>1 , utilizes a approach of [13] to global stability problem in R^n.

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